Mathematics
Higher level
Paper 3 – sets, relations and groups

Thursday 15 November 2018 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 13]

Consider the binary operation $*$ defined on the set of rational numbers, $\mathbb{Q}$ by $a*b = a + b + ab$.

(a) Show that the operation $*$ is

(i) commutative;

(ii) associative. [5]

(b) Solve the equation $a*b = -1$. [3]

Let $S = \{x \in \mathbb{Q} | x \neq -1\}$.

(c) Show that $\{S, *\}$ is an Abelian group. [5]

2. [Maximum mark: 8]

Consider two subsets X and Y of a universal set U.

(a) Use De Morgan's laws to prove that $\left[\left(X' \cup Y'\right)' \cap Y'\right]' = U$. [4]

Let $X = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} \mid m = n^2\}$ and $Y = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} \mid 0 \le m + n \le 6\}$.

(b) List all elements of $X \cap Y$. [4]

3. [Maximum mark: 10]

A relation R is defined on $\mathbb{R} \times \mathbb{R}$ by $(a, b)R(c, d) \Leftrightarrow 3(a - c) = 2(b - d)$.

(a) Show that R is an equivalence relation. [8]

(b) Find the equivalence class containing $(1, 2)$ and describe it geometrically. [2]

4. [Maximum mark: 9]

Consider the functions $f, g : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ defined by

$$f((x, y)) = (x + y, x - y) \text{ and } g((x, y)) = (xy, x + y).$$

(a) Find

(i) $(f \circ g)((x, y))$;

(ii) $(g \circ f)((x, y))$. [5]

(b) State with a reason whether or not f and g commute. [1]

(c) Find the inverse of f. [3]

5. [Maximum mark: 10]

Consider a finite group $\{G, *\}$. Let H be a subgroup of G of order m such that $G \setminus H \neq \varnothing$. Let a be a fixed element of $G \setminus H$. Consider the set $A = \{a*h \mid h \in H\}$.

(a) Show that $A \cap H = \varnothing$. [3]

Consider the function $f : H \to A$ defined by $f(h) = a*h$.

(b) Show that f is a bijection. [4]

(c) Find the number of elements in the set $A \cup H$. [3]